Quiz 3.1: Sample Answers

Note: beginning with these set of questions, we can now use the differentation rules we have learned.

1. Differentiate $f(x) = -2e^x - \sqrt{x}$

   The function can be re-written as $f(x) = -2e^x - x^{1/2}$. We can then use our rules of differentation to get

   $$f'(x) = -2e^x - (1/2)x^{-1/2} = -2e^x - \frac{1}{2\sqrt{x}}$$

2. For $f(x) = x^2 + 4e^x$, find the equation of the tangent line at $P = (0, 4)$.

   We first find the derivative:

   $$f'(x) = 2x + 4e^x$$

   Then substitute $x = 0$ to get the slope:

   $$f'(0) = 2(0) + 4e^0 = 0 + 4 = 4$$

   We have $m = 4$, and the point $(x, y) = (0, 4)$. We then substitute these into $y = mx + b$ to solve for b:

   $$(4) = (4)(0) + b \Rightarrow b = 4$$

   So the equation is $y = 4x + 4$.

3. For $f(x) = 2x^3 + 12x^2 + 18x + 12$, where is its tangent line horizontal?

   To get a horizontal tangent line, we need the slope to be 0, so we need to find where $f'(x) = 0$.

   $$\begin{aligned} f'(x) &= 0 \\ 6x^2 + 24x + 18 &= 0 \\ x^2 + 4x + 3 &= 0 \\ (x+1)(x+3) &= 0 \end{aligned}$$

   Thus at $x = -1$ or $x = -3$ we have a horizontal tangent line.

4. Find where $f(x) = x^3 + 6x^2 + 12x + 3$ has tangent line with slope 24.

Since slope of the tangent line at x is $f'(x)$, we need to set $f'(x) = 24$.

$$\begin{aligned} f'(x) &= 24 \\ 3x^2 + 12x + 12 &= 24 \\ 3x^2 + 12x - 12 &= 0 \\ x^2 + 4x - 4 &= 0 \end{aligned}$$

This polynomial can not easily be factored, so we must use the quadratic formula, which gives

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

for the polynomial $ax^2 + bx + c = 0$. Here, we have $a = 1$, $b = 4$, $c = -4$, so we get

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-4)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16 + 16}}{2}$$

$$x = \frac{-4 \pm \sqrt{32}}{2}$$

$$x = \frac{-4 \pm 4\sqrt{2}}{2}$$

$$x = -2 \pm 2\sqrt{2}$$

So the solutions are $-2 + 2\sqrt{2}$ and $-2 - 2\sqrt{2}$.